

# LENS DISTORTION SIMULATION. AN APPLICATION FOR UNDERSTANDING GEOMETRIC DISTORTION.

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### ABSTRACT:

The popularisation of photography as a documentary basis in the field of architectural and archaeological heritage recording, sustained by the development of more and more powerful computers and easy-to-use software tools, has brought as a consequence the popularisation of Photogrammetry.

Simultaneously its happening the explosion of a technology phenomenon with positive and negative effects: The digital camera. Day by day they are becoming more capable, with higher resolution, smaller and cheaper. They are, without any doubt, the unavoidable successors of film cameras at any level.

The new image sensors are being vastly and unceasingly improved. The professional branch evolves in the sense of raising the sizes up to the standard 36 x 24 mm with resolutions of over 10 Mpix, nearly able to fit into the usual SLR bodies but sadly by now at very high prizes. At the same time, in the consumer market, the sensors are being improved and becoming cheaper at sizes of about 1cm. For these, very small lens groups are needed, and for them almost microscopic focus mechanisms. This implies that high geometric image quality is very difficult to achieve.

Even the best semi-professional digital cameras show very noticeable lens distortion and the images need to be corrected and thus resampled in order to have all of their potential benefits in the photogrammetric usage and particularly if they are taken for rectification purposes.

The lens geometric distortion effects and their relation with the various mathematic expressions used for its characterisation, seems often too abstract for non-skilled users or better, for users that have not specific background in Optics, or Photogrammetry. For this reason, we have developed an application that is called LDS (by means of Lens Distortion Simulator) that we present as a tool for experimentation, simulation and correction of the lens geometric distortions in digital imagery. It is particularly oriented to academic scenarios and, in a more general point sense, to users and professional of Photogrammetry.

## 1. INTRODUCTION

The progressive computerization of technologies, particularly since 1996, when digital photogrammetric workstations became well-settled, and with the boom of the World Wide Web, have given rise to the popularisation of Photogrammetry.

We consider that this trend finds its *raison d'être* and its better development space within the CIPA context. Lets highlight several facts that particularly sustain this appreciation.

- The use of sensors that are more powerful and affordable day by day and thus well-suited for heritage recording purposes. (WG7).
- The development and improvement of surveying methods that, with different accuracy and efficiency levels, have been applied to diverse fields embracing as different subjects as satellite imagery and tactile methods (WG3, WG6).
- The availability of data processing tools have become more open, hybrid and less classic day by day : Today's photogrammetric software tends to keep the General Method (that stands for the classic scheme of inner–relative–absolute orientations) away from the user and dissolved by the implementation of userfriendly interfaces. The same happens whith image processing and

computer graphics (CAD) which are becoming more effective to render heritage objects (WG3, WG4).

- The creation of meeting and discussion forums where different specialists can converge (bridge the gap) to a common objective (WG1, WG2).
- The explicit impulse given to the “Photogrammetry for everyone”, which is best represented by proposals such as the “3x3 rules” (1988 Brunner-Waldhäusl) that helps non-specialized people to get involved in heritage recording. (TG1).
- The progressive rapprochement to photogrammetric concepts by an increasing number of people, thanks to the development of e-learning tools and the generalised Internet access.
- The constant development of new applications to suit specific objectives is being possible thanks to the availability of easy to use and learn object oriented programming languages. These have allowed the user to customize his workspace and tools for each need.

Within this panorama, our professional and teaching experience brings us many times to ask ourselves one question: Under what circumstances could we make use of non- calibrated cameras for heritage recording purposes?.

In this paper we want to focus on three aspects concerning this question:

1. How do we manage with the high degree of geometric distortions (particularly the radial symmetric distortion) when we apply direct image resampling techniques such as rectification?
2. How will we set up internal camera parameters including those related to lens distortions when we make use of different programs? And being more specific, how will we manage all the different mathematic schemes that are involved in the description of the same phenomenon?
3. In case of being the distortion well-known, what is the most efficient way of working? Should be a good practice to eliminate it as a first stage by image re-sampling prior to any further process such as re-projection or restitution? Or would it be better to have it mathematically modelled doing the re-sampling as a unique stage that maps for instance distortion and rectification at the same.

It is obvious that, nowadays, we are more focused on a certain type of cameras, those that have been leading the reconciliation of non-skilled people and Photogrammetry. In a general sense, the boom of this science as a powerful instrument for heritage recording has been made possible thanks to the irruption of digital cameras on the scene.

## 2. INTRODUCING “LDS”

Lens Distortion Simulator is a computer application that will bring some light to some points that are often found obscure by many users, and particularly by students when they face this question. The following lines will act as a review of some key points. We will just focus on the radial symmetric distortion and not so much on the tangential and asymmetric due to the higher dimensional magnitude and conceptual importance of the first one as it affects the principal distance concept. For this reason, only radial distortion simulation is being implemented with the main purpose of simulating its consequences on images; this is something that we consider very didactic. On the other hand, it will allow to know which range applies to those parameters and at what levels they have noticeable effects.

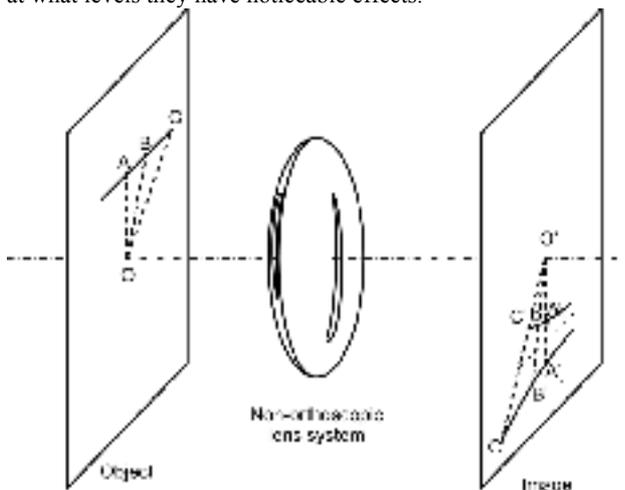


Figure 1. The simple fact.

In the classic Optics language, as shows the figure 1, the distortion is defined as the inconstancy of lateral magnification. But even if the classic definition is easy to learn, as it explains the differences in terms of point image-coordinates between real and theoretical locations (those resulting from the

perspective laws compliance), the complexity comes from the diverse ways of expression of this difference sometimes as an error and sometimes as a correction. (It depends on authoring). These displacements, that can be easily understood, become hard, dense and opaque when the user finds that different programs use different nomenclature and parameterisation. As a result sometimes the user finds expressions of error while in others terms of corrections. But all definitions are in fact the same; all models define the displacement of points from their bundle perspective rules compliant positions, but using different notation to this same fact.

## 3. THE CONCEPT OF RADIAL SYMMETRIC DISTORTION.

It is important to take into consideration that the concept of radial distortion itself does not offer a clear panorama at all, at least from a didactic point of view

Many authors such as Bonneval, Ghosh, Moffit, or Mikhail, define radial distortion as the deviation of light rays during lens crossing.

It seems that such definition comes from the didactic need of making the model fit into a perfect projective scheme where the projection centre can be exactly located in a certain point (like pinhole camera), or instead of this, it could also be considering the nodal image point being equally well-located and determinable.

In this way, the lense’s radial distortion is measured as a distance or separation (along radial directions contained in the image plane) between the actual positions and their ideal corresponding ones. So that an ideal ray trajectory and its resulting image spot are exactly determined by the value of the principal distance ( $f$ ) and the incidence angle ( $\alpha$ ).

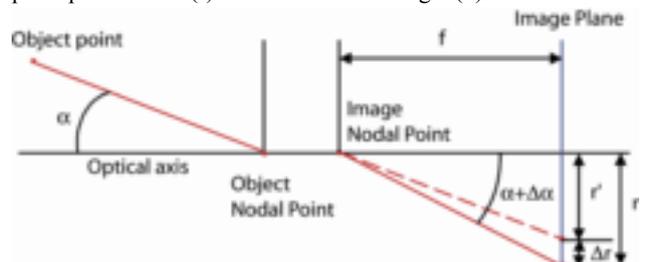


Figure 2. Scheme of ray deviation.

Under such assumptions, the radial distortion is expressed in terms of residuals or errors: actual position – theoretical position.

$$dr = r - r' = r - f \cdot \tan \alpha \quad [1]$$

This expression of “error” gives us the sign criterion: It will be considered positive the outgoing way and negative the opposite one. We can see its results in the following figure: The red figure corresponds to a positive distorted image (“pincushion”) of a perfect square shape, while the blue one results from a negative distortion (“barrel”).

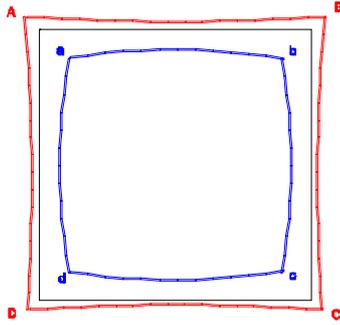


Fig. 3. Barrel (blue) and pincusion (red).

This formulation could make the student think of “principal distance” as an invariable parameter. The great importance given to the focal length inside the photogrammetry context contributes to increase this risk. The focal length plays, without any doubt, a leading role not only in mathematic models such as co-linearity condition or co-planarity, but also during setting up of a stereoplotter and in photogrammetric projects planning. The high accuracy that, as it is assumed, underlies in a calibration certificate (in which principal distance is usually expressed in a magnitude order of microns) highlights the pre-eminence of focal length as the most basic parameter in Photogrammetry.

But being true that the principal length value must be unique, one must notice that this parameter is directly correlated to other parameters and together they define the internal characteristics of the camera, so that a change or lock of anyone of them must have effects on the others.

The problem, as we all known, is that while one can observe and thus measure the value of the incidence angle (alpha) and the radial distance resulting from it (r), it is impossible to measure the principal distance. On the contrary, we infer the knowledge of the principal length from what we want to know: the radial distortion that has already been defined as a function of the same distance. So nothing is that well-defined.

The principal length is the distance between the image nodal point and the image plane which is located in a certain point where both the actual and the theoretical image-points are the same. But, in fact, we can't neither know at what distance that occurs (where radial distortion is null), nor find it useful in operative terms. In any case an additional criterion is needed. As Brown says there are three possibilities.

- a) We can assume that the principal distance makes null the radial distortion at a fixed radial distance R0
- b) We can solve for a principal distance that makes minimum the summation of squares of deviations.
- c) Or we can search for the principal length that makes equal absolute values of maximum and minimum deviation.

Another way of talking about radial distortion has been used by Albertz, Kraus or Burnside. For them, the radial distortion can be considered as the variation of the principal distance, as a function of the incidence angle of rays (we do prefer this version due to its didactic value). This distance should be given as a nominal value by the calibrator so the assumption of a fixed physical dimension is better avoided. In this way it is understood that this parameter depends on a predefined

specification and gives as a result an specific distribution of radial distortion values.

$$r = (f + \Delta f) * \tan \alpha$$

Following Hallert's notation

[2]

that we rather prefer instead of the more commonly seen:

$$r = f * \tan(\alpha + \Delta \alpha)$$

[3]

Even when the nodal point actually exists, and in the image plane there is a certain region of points that are perspective rules compliant, it is useless to search for their position. The reason of this is that any elected value of the principal distance is a good choice if it is taken into account that the discrepancy the actual principal distance and the preset one. This discrepancy brings on a certain distribution of the displacement ( $\Delta r$ ) of every image point from its ideal radius ( $r'$ ) as a consequence.

$$r / f = r' / f'; \Delta r = \Delta f r' / f'$$

[4]

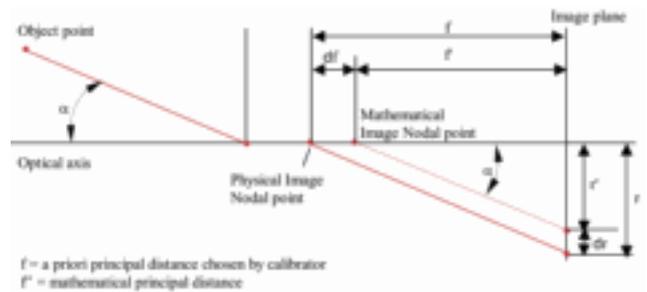


Figure 4. Scheme of interdependence between  $f$  and  $dr$ .

From a practical point of view, there is no need to use the concept of “true principal distance”, but to know the diverse distribution of radial distortion ( $dr_1, dr_2, \dots, dr_i$ ) associated to their corresponding focal lengths ( $f'_1, f'_2, \dots, f'_i$ ) as functions of incidence angle.

$$\frac{r_i}{f_i} = \frac{r - dr_1}{f'_1} = \frac{r - dr_2}{f'_2} = \frac{r - dr_i}{f'_i} = \frac{r_i + \Delta r_i}{f'_i}$$

[5]

What we pretend to show is that a given physical point always has the same image point, associated with a residual that function of its distance to the principal point (best symmetry point). In other words: for a certain object point coordinates, there only exists a unique pair of image coordinates which corresponds to it (orientations are supposed fixed) but there are virtually infinite combinations of principal length and distortion that make colinearity condition complied.

The application that we have developed offers a workspace in which the user can simulate the effects of radial distortion on a test pattern, seeing its connection with the focal length. It emulates in some way a multi-collimator; a grid of points project light rays through a virtual lens (the lens axis is supposed to be normal to the grid plane or  $w = f = 0$ ). Fig. 5.

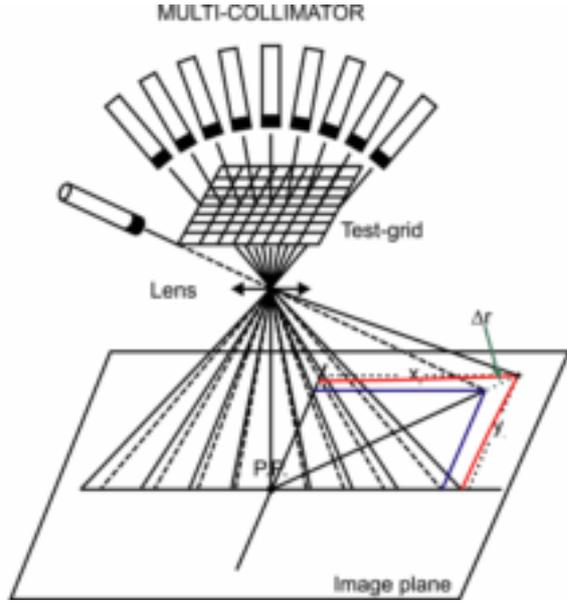


Figure 5. The grid simulates a collimator.

The mathematic model is as follows:

$$\begin{aligned} (x-x_0)(1+a_1r^2+a_2r^4+a_3r^6) &= -f \frac{r_{11}(X-X_s)+r_{12}(Y-Y_s)+r_{13}(Z-Z_s)}{r_{31}(X-X_s)+r_{32}(Y-Y_s)+r_{33}(Z-Z_s)} \\ (y-y_0)(1+a_1r^2+a_2r^4+a_3r^6) &= -f \frac{r_{21}(X-X_s)+r_{22}(Y-Y_s)+r_{23}(Z-Z_s)}{r_{31}(X-X_s)+r_{32}(Y-Y_s)+r_{33}(Z-Z_s)} \\ r &= \sqrt{(x-x_0)^2+(y-y_0)^2} \end{aligned} \quad [6]$$

where the following is assumed.

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = 0 \quad ; \quad \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = 0 \quad ; \quad \begin{bmatrix} \omega \\ \phi \\ \chi \end{bmatrix} = 0 \quad [7]$$

where the coordinates  $(X, Y)_1$  of the grid nodes being  $Z_1$ . So that the model can be written as below:

$$\begin{aligned} x_i &= \frac{-f}{Z(1+a_1r^2+a_2r^4+a_3r^6)} X_I \\ y_i &= \frac{-f}{Z(1+a_1r^2+a_2r^4+a_3r^6)} Y_I \\ r &= \sqrt{(x_i-x_0)^2+(y_i-y_0)^2} \end{aligned} \quad [8]$$

where  $(x_i, y_i)$  are image coordinates affected by radial distortion.

Once the expressions of the image coordinates have been isolated, it is seen that in both right side terms there are unknowns: the distorted coordinates. As long as we have set up perfect coordinates as a starting for simulating their distortions, it is needed to use an iterative strategy to evaluate that system.

The user is allowed to modify the four basic parameters ( $a_1, a_2, a_3, f$ ) and evaluate the effect of their changes. It's also possible to arrange a grid of image points just simulating any taken

picture, and see how distortion distribution changes according to the  $f$  variations set by user.

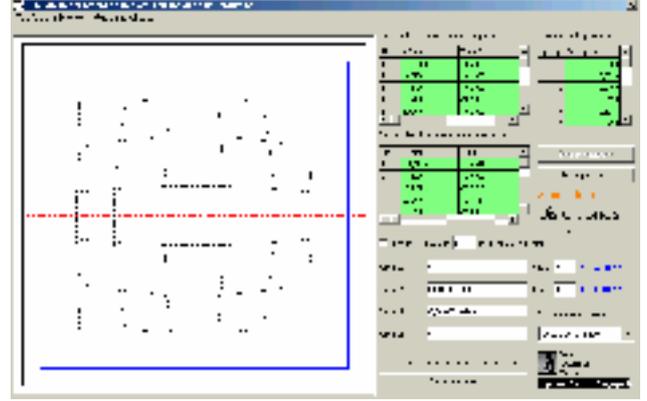


Figure 6. LDS's user interface.

The effects of related variations are graphically shown and will help the user to understand the distortions appearing on images.

#### 4. RADIAL DISTORTION MODELS.

We have already pointed out to some difficulties concerning this problem. In fact, there is not a unique model to explain the relationship between the radius and the radial displacement.

Distortion correction:

$$\Delta r = F(r) \quad [9]$$

Radial distortion value:

$$dr = F(r) \quad [10]$$

1. Brown's model:

$$-dr = \Delta r = a_1r^3 + a_2r^5 + a_3r^7 \quad [11]$$

2. USGS's model:

$$-dr = \Delta r = a_0r + a_1r^3 + a_2r^5 + a_3r^7 \quad [12]$$

3. ISPRS's model:

$$-dr = \Delta r = a_1r(r^2 - r_0^2) + a_2r(r^4 - r_0^4) + a_3r(r^6 - r_0^6) \quad [13]$$

Notice that the third one can easily be identified with the USGS's if the following is assumed:

$$-a_0 = r_0^2 * (a_1 + a_2 * r_0^2) \quad [14]$$

As it is said before, while the distorted model remains the same, any change in focal length brings about a new distortion scheme (distribution) and also new coefficients ( $a_i$ ) as a consequence:

$$\frac{a_{1j}}{f_1} = \frac{a_{2j}}{f_2} = \frac{a_{ij}}{f_i} \quad [15]$$

In LDS special attention has been paid to the comparison between Gaussian model of distortion correction versus that so called balanced one (ISPRS) due to the meaning of the last one.

Lets reformulate the first expression above to avoid the confusion caused by the use of the same letters for the coefficients:

$$\Delta r = k_1 r^3 + k_2 r^5 + k_3 r^7 \quad [16]$$

On the other hand.

$$\Delta r = a_1 r(r^2 - r_0^2) + a_2 r(r^4 - r_0^4) + a_3 r(r^6 - r_0^6) \quad [17]$$

that is equivalent to

$$\Delta r = a_0 r + a_1 r^3 + a_2 r^5 + a_3 r^7 \quad [18]$$

See in the figure 7, how the line  $\Delta r = a_0 r$  serves to have the plot of

$$\Delta r = k_1 r^3 + k_2 r^5 + k_3 r^7 \quad [19]$$

in the form of this one:

$$\Delta r = a_0 r + a_1 r^3 + a_2 r^5 + a_3 r^7 \quad [20]$$

The slope could be freely chosen in order to fit some needs, as it is for instance, to have the values of distortion arranged in a weight-balanced distribution. (equal areas of positive and negative distortion values or equal maximum and minimum). At the second zero of the curve (see the expression above) we can write  $a_0 = -a_1 r_0^2 - a_2 r_0^4$

So the same expression could be rewritten as follows:

$$\Delta r = (-a_1 r_0^2 - a_2 r_0^4) r + a_1 r^3 + a_2 r^5 = a_1 r(r^2 - r_0^2) + a_2 r(r^4 - r_0^4) \quad [21]$$

Then, lets call the principal distance corresponding to the Gaussian graph  $f_g$

$$\Delta r_g = k_1 \cdot r^3 + k_2 \cdot r^5 \quad [22]$$

and  $f_b$  the same referred to the *balanced* form:

$$\Delta r_b = a_0 \cdot r + a_1 \cdot r^3 + a_2 \cdot r^5 \quad [23]$$

Lets notice that both families must comply the same relations.

$$\frac{r + \Delta r_g}{f_g} = \frac{r + \Delta r_b}{f_b} = \frac{r + k_1 r^3 + k_2 r^5}{f_g} = \frac{(1 + a_0) r + a_1 r^3 + a_2 r^5}{f_b} \quad [24]$$

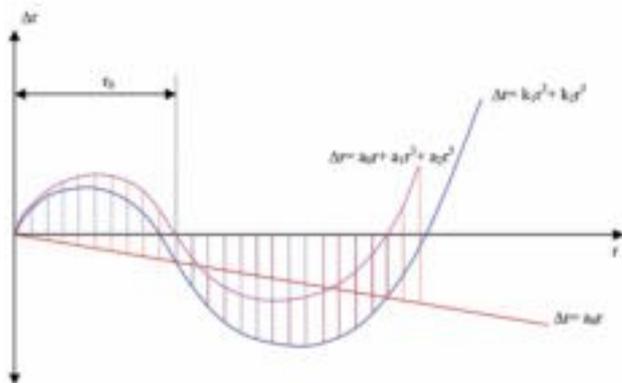


Figure 7. Radius / Distortion graph.

Therefore:

$$\frac{f_b}{f_g(1 + a_0)} = \frac{r + \frac{a_1}{1 + a_0} r^3 + \frac{a_2}{1 + a_0} r^5}{r + k_1 r^3 + k_2 r^5} \quad [25]$$

that brings us

to conclude that:

$$\begin{aligned} f_b &= f_g (1 + a_0) \\ a_1 &= k_1 (1 + a_0) \\ a_2 &= k_2 (1 + a_0) \end{aligned} \quad [26]$$

Our simulator allows the user to see how the Gaussian and balanced forms are related one to each other, showing equivalences between all the implied parameters in real time..

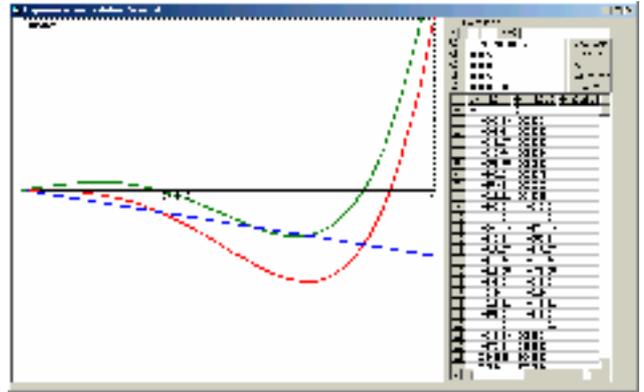


Figure 8. LDS's Distortion graphing intergace.

## 5. SIMULATION RESULTS.

Once arranged all the mathematic models, we have tried various settings for testing several inner orientations for virtual cameras. After that, we have rendered the results to three-dimensional graphic outputs with the purpose of being of help for users to understand the models. (These plottings have been created with Matlab®)

The figures below are 2d and 3d plots representing a typical distortion distribution. Colour gradation enables a better visualisation of the shape and can also be intentionally set up to highlight zones where distortion surpasses usable levels.

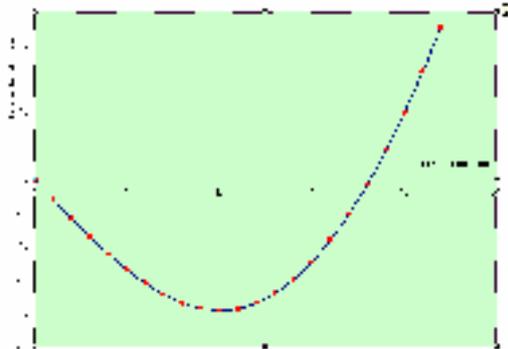


Figure 9. Distortion curve report.

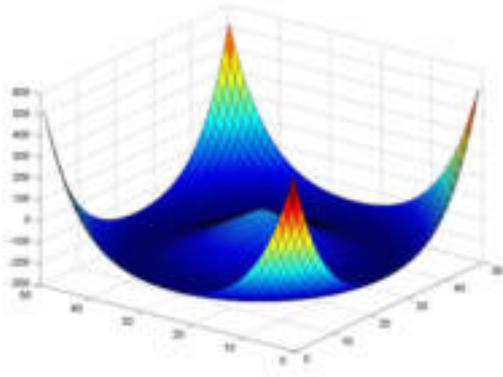


Figure 10. Three-dimensional distortion plot.

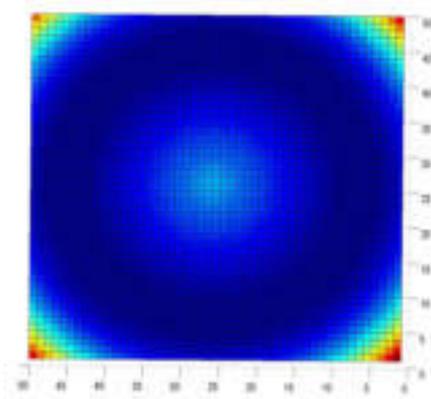


Figure 11. Distortion colour map.

We could simulate as well the effect of distortion over a point pattern. That will make us feel familiar to the resulting scheme of distortion with regard to the sign and magnitude of the parameters that come into play.

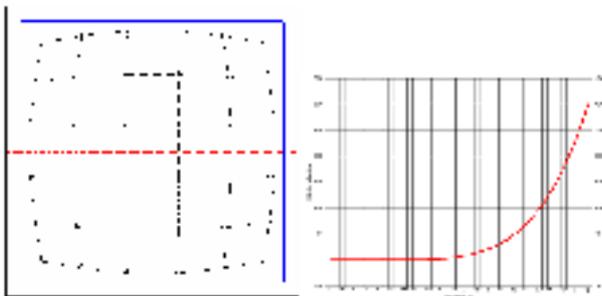


Fig. 12. Distorted grid and Ms-Excel ® Plot.

With the same purpose of making further analysis easy, the user is allowed to copy any of the numerical results as single data or as tables as well, so that it can be, for instance, pasted in a worksheet.

One of the goals of LDS is to allow us to test any set of parameters over an image to resample it and save as a new one showing the desired distortion, this feature will be useful to get ortho-scopic images (distortion free images) if the lens parameterisation is already known.



Fig. 13. Mariana. (original picture)



Fig. 14. She again. (corrected picture)

One more possible application of LDS could be the creation of pure distorted images from a virtual scene drafted within a CAD program. These images processed by LDS can emulate a perfectly distorted photograph, and thus serve as test-dummies for other photogrammetric software development (such as stereo digitizers) saved from errors derived from badly calibrated cameras.

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