

GEOMETRIC OPERATIONS IN DIGITAL IMAGES

Carlo Monti*, Francesco Guerra**, Caterina Balletti***, Davide Miniutti****

* Full Professor

**3rd year Course of PHD

***2nd year Course of PHD

****1st year Course of PHD

DIAR Dipartimento di Ingegneria Idraulica, Ambientale e del Rilevamento

Sezione Rilevamento - Politecnico di Milano

P.zza Leonardo da Vinci 32 – 20133 Milano

ITALY

e-mail: direz@idra5.iar.polimi.it

guerra2@brezza.iuav.unive.it

balletti@brezza.iuav.unive.it

miniutti@cidoc.iuav.unive.it

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ABSTRACT

The geometrical transformations change the spatial relationship between objects in an image. One of the principle applications of geometric transformations concerns the possibility of correcting the digital images of the distortions introduced by the camera. Often the images produced by inexpensive cameras present geometric distortions of a very large nature. These images must be corrected in order to be used metrically in the survey. The traditional approach to the problem is to modellize the aberrations (distortions) of the lens system and correct the images obtained. In the approach utilised in the work that is presented here, only the effects on the final image are evaluated, without researching (and therefore modellizing) the factors that have caused these effects.

Also introduced here are the experiences earned by applying a calibration method based on the application of a polynomial warp usually named rubber sheeting. This method has been applied experimentally to some non professional digital photocameras whose internal orientation, with infinite focus, can be considered to be stable. The method is based on the shot of a measured control network and on the estimation of the parameters of the polynomial on the basis of the points measured on the image and on the network. A specific software has been realised for the implementation of the algorithm which will be heretofore introduced and described.

This method was also tested on non metrical 35 mm format cameras and objectives with strong distortions (fisheye lens). The geometrically corrected images can be utilised for photogrammetrical applications, GIS and multidata integration.

GEOMETRIC DISTORTIONS

When non metric cameras are used for application of terrestrial photogrammetry, the distortions constitute the type of optical aberration which is greatly feared.

The use of an amateur type of camera, especially with lenses with an elevated level of distortion, pose problems which are not minor ones, given that the representations of the photographed object which are obtained are not of central perspectives; the deformations are not to be neglected and in order to use the images metrically, they must be corrected.

The lens of the photographic cameras introduce two types of deformations:

- radial
- tangential

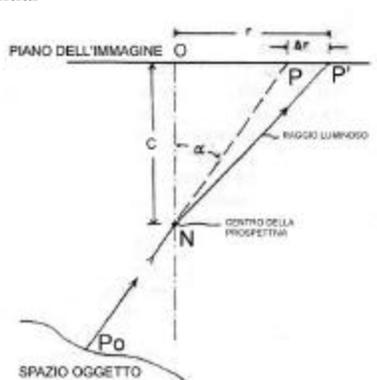


fig.1

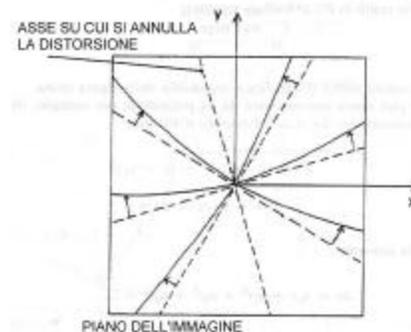


fig.2

The effects that are provoked on the photographs of an object are illustrated in the *fig.1*, from which it can be deduced that the effect of the radial distortion of the image on a point P_0 , rather than shape itself into P , as the gaussian lens would have, shapes itself into P' , and in *fig.2* which schematises the effects of the tangential deformation.

While in the metrical cameras the problem is resolved upstream, in the amateur cameras, the problem is usually overcome through the elaboration of a mathematical model which approximates the corrections to be applied.

The software elaborated by the authors and proposed in this paper utilises another type of approach: the use of mathematical equations, the polynomials known as rubber sheeting, which permits the correction of the images without having to investigate the causes that bring about the deformations.

DIGITAL IMAGES

In order to test the algorithm implemented, two digital cameras and two traditional cameras were used; for these traditional cameras, the photographs were digitalised successive to the test with the use of a scanner.

The use of digital images is fundamental in so much as it allows the easy use of the typical spatial analysis elaboration. Presumed for such operations is that the digital images have been returned to a known geometry, utilising the specialised procedures of referencing and calibration.

The software presented resolves a part of these problems by using a polynomial plane transformation whose number of parameters is chosen by the user.

THE TRANSFORMATION ALGORITHM

For the realisation of this software the most generic form of the plane transformation was used extracting the transformations usually utilised as a particular case of the utilised transformation.

The general equation which regulates the plane transformation between two systems of coordinates (in the specific case, whole pixel coordinates and coordinates of the external reference system) is represented by a n order polynomial such as:

$$X = \sum_{i=0}^m \sum_{j=0}^n a_{ij} x^i y^j$$

$$Y = \sum_{i=0}^m \sum_{j=0}^n b_{ij} x^i y^j$$

The procedures which utilise this polynomial are noted with the name *rubber-sheeting*; at times in the application packages (GIS or CAD) they are referred to with the more generic term of *warp-transformation*.

If the polynomial is developed, the following is obtained:

$$X = a_{00} + a_{01}y + a_{10}x + a_{11}xy + a_{20}x^2 + a_{21}xy^2 + a_{22}x^2y^2 + \dots$$

$$Y = b_{00} + b_{01}y + b_{10}x + b_{11}xy + b_{20}x^2 + b_{21}xy^2 + b_{22}x^2y^2 + \dots$$

As can be easily seen, the transformations commonly used in the survey discipline are obtainable considering the first terms of the developed polynomial.

The similar transformation (rototranslation with variations of scale) is obtained by the first 6 terms:

$$X = a_{00} + a_{01}y + a_{10}x$$

$$Y = b_{00} + b_{01}y + b_{10}x$$

where

$$a_{01} = b_{10} \quad a_{10} = -b_{01}$$

so the independent parameters are only 4.

The same transformation is obtained by the first 6 without any conditions:

$$X = a_{00} + a_{01}y + a_{10}x$$

$$Y = b_{00} + b_{01}y + b_{10}x$$

the bilinear transformation by the first 8:

$$X = a_{00} + a_{01}y + a_{10}x + a_{11}yx$$

$$Y = b_{00} + b_{01}y + b_{10}x + b_{11}yx$$

It would seem evident that acting on the number of the coefficients it is possible to utilise the program for typical applications in the survey and in the representation as an internal orientation of the photograms, the rectification, texture mapping, etc. In this sense, the software offers the same operative possibilities of other software for the geometric treatment of the images.

In the case in which more than 8 coefficients are used, it is important to make some observations. The equations are satisfied exactly if the number of points known $n = \frac{1}{2} p$, with the p number of the parameters taken into consideration. It seems evident, therefore, that if there is a large number of points known and if an exact solution is wanted then higher grade polynomials must be used. In general, it is better to use a larger number of points than what is strictly necessary in order to:

- have an estimate in least squares of the parameters of the transformation to evaluate the results of the transformation;
- avoid the problem of the over parameterisation.

This second aspect is particularly important insofar as often an excessive number of parameters, while allowing for a better solution from the numerical point of view (with equal control points, lesser residuals are obtained) may bring about a description of the phenomenon of transformation far from the physical reality.

On the basis of a set of points, the GPT package allows for the geometric referencing and transformation of the images according to their utilisation in the GIS.

The control points are measured on the image (indices i, j) and supplied to the reference system into which the image must be referenced. The numerousness of the points is variable according to the geometric transformation which is hoped to be adopted and the redundancy which is hoped to be obtained in the estimation to least squares of the parameters of transformation.

The programs allows for the choice:

- of the type of transformation
- of the number of parameters to use (or alternatively, the degree of the polynomial);
- of the dimension of the pixel in the external reference system;
- of the type of algorithm for the interpolation (closest, bilinear, bicubic)).

As a release the program offers:

- resampled image, with the axes parallel to the reference system and with the pixels in the chosen dimension;
- two files with the parameters of georeferencing

An interesting characteristic of this program is that of the superimposition of the vectors of the residuals on the image which allows for the easy individuation of the zones where eventual errors might be present.

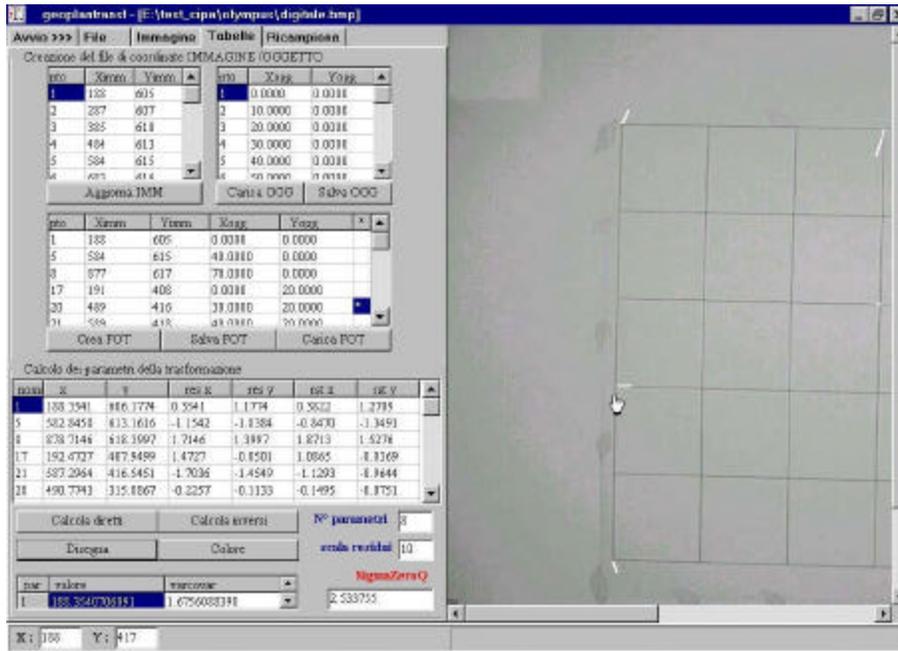


fig.3 - GeoPlanTrans program: window for the choice of the control points and the superimposition of the vectors of the remainder onto the image

RESAMPLING

For the creation of the final image, one having axes parallel to the external reference system, the technique of *inverse mapping* is used, which consists of filling an appropriately dimensioned empty image with values obtained by interpolation of the original image.

The algorithms used for the resampling work according to three methods: nearest neighbour assignment, bilinear interpolation or bicubic splines.

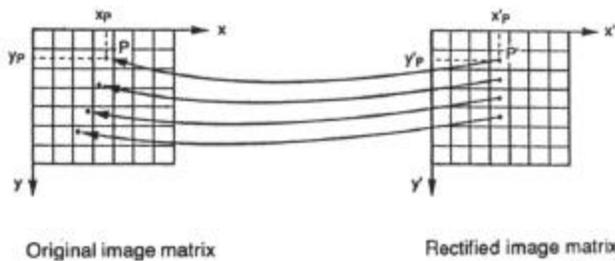


fig. 4: the inverse method of rectification

The formula used by the zero order interpolation (nearest neighbour) is:

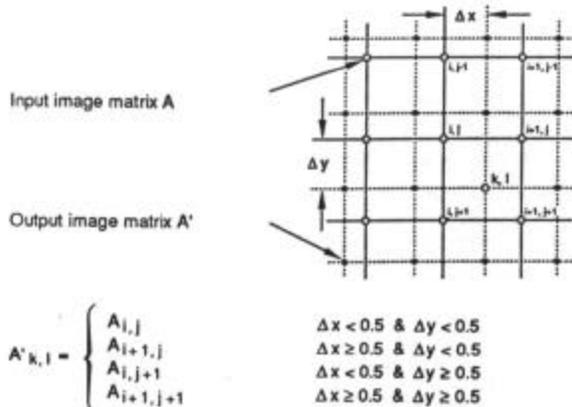


fig. 5: resampling methods: zero order interpolation

The formula used by the bilinear interpolation is:

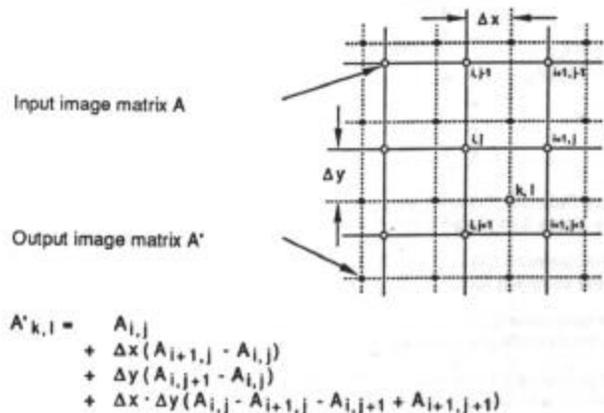


fig. 6: resampling methods: bilinear interpolation

The formula used by the bilinear interpolation is:

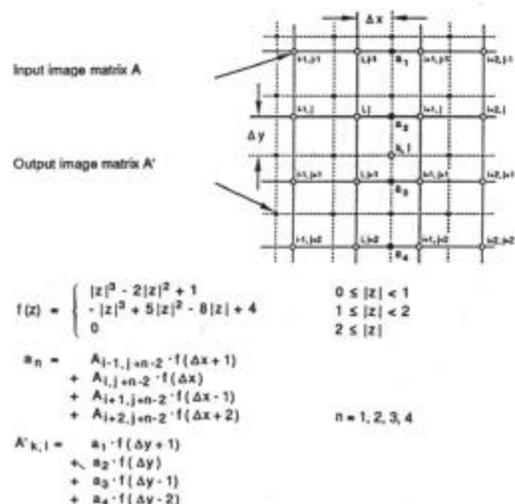


fig. 7: resampling methods: bicubic spline interpolation

THE PROCEDURE

The test is based on the capturing of a reticule composed of 7x5 quadrants with side $l = 10$ cm (fig.9)

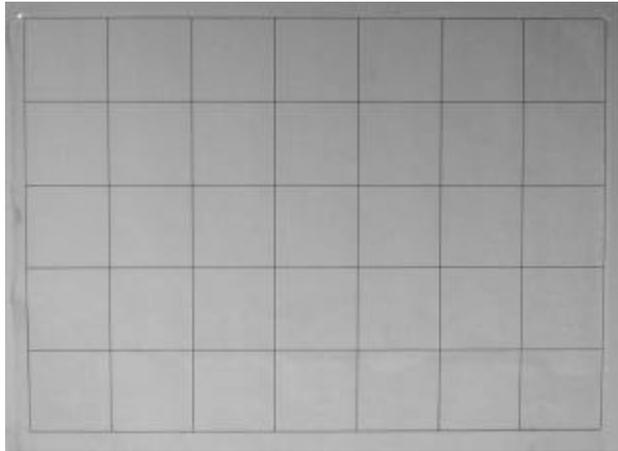


fig.9

The photographs have been taken with the optical axes perpendicular to the plane of the reference reticule. If this case was not possible, a 8 parameters transformation (homography) to eliminate the effects of the perspective. In this kind of transformation, in fact, straight lines remain straight, therefore the effects caused by the distortions of the lens are maintained. An estimation of the parameters of the polynomial form is done following the method described below; parameters can be archived and utilized afterwards for the camera calibration.

The procedure can be summarized in the following steps:

- photographs of the reference grid;
- 8 parameters transformation to eliminate the effects of the perspective;
- estimation of the parameters of the polynomial transformation.

TEST

As it has been mentioned, the application of the program has been done by utilising the two digital cameras (an Olympus Camedia 35 mm with 1024x768 resolution and a Canon Power Shot Pro70 28mm with 1536x1024 resolution) and two non metrical reflex (one with a Pentax 28mm lens and the other with a Tamron 17mm lens)



Canon Pro70



Olympus Camedia

For each camera, many geometric transformations were effected, varying each time the number of control points and the number of the parameters in order to see what influence these two variables have on the transformation itself. In each case, the number n of the known points was always maintained greater than the minimum number necessary to satisfy the equations

$$n > 1/2 p, \text{ with } p \text{ the number of parameters}$$

this in order to evaluate the results of the transformation through an estimate to the least squares. As far as the resampling is concerned of the corrected images, bilinear interpolation was chosen.

In the case of the two digital cameras and for the approximate frontal settings, the results have been analogous: as can be observed from the superimposition of the transformed images with the vectorial reticule (fig.10) the distortions of the photograph, in this case 20 control points are used, (that is to say, a rather high number of points) diminish with the increase in the number of parameters used and therefore with the increase in the degree of the polynomial; when 20 parameters have been surpassed, however, (polynomial of the third degree) the situations worsens: as has been said, an excessive number of parameters allows for a better numerical solution to the problem but offers a description of the deformation quite different from the physical reality. The (fig.11) underlines in fact how the remainders of the observations diminish with the increase in the numbers of parameters of the polynomial. With these two cameras, analogous results have been obtained utilising a lesser number of points (10), providing that, for obvious reasons, these are distributed in a reasonable manner, which is to say, at the edges of the image.

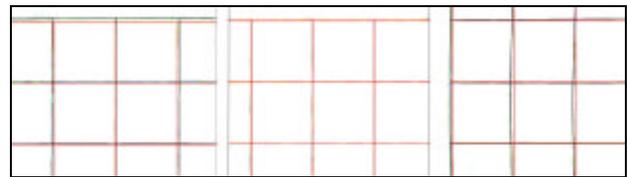
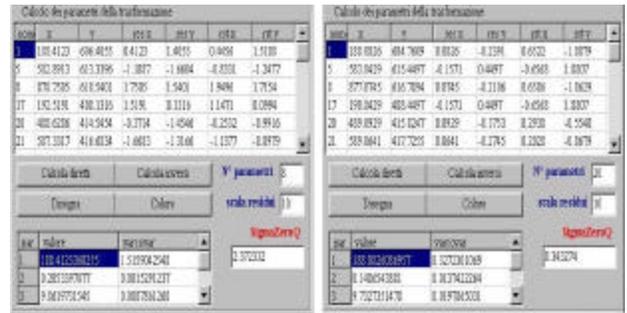


fig.10 - a)20 points with 8 parameters, b)20 points with 20 parameters, c)20 points with 30 parameters



(a)

(b)

fig.11 - residual calculation in the case of development with 10(a) and with 20 (b) parameters

Very different still are the results obtained in the case that the photograph is very foreshortened: the resampling of the images does not bring about acceptable results as in the preceding case and the deformations are that much more amplified as there are even a lesser number of control points available. Only in the case in which a very high number of points is used and distributed all along the surface of the image are acceptable results achieved (fig.12).

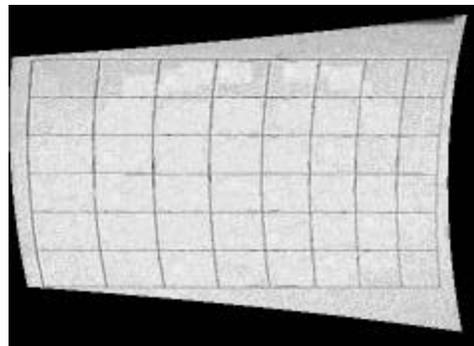


fig.12

For images with a certain geometry, it can be possible to make an estimated rectification utilising, for example, four points and then to correct the deformations as in the preceding case through polynomial transformation.

The tests done with the traditional style camera have given completely different results according to the lens used: if as in the case of the 17mm, the results seem to be terrible, no matter what the number of control points or the number of parameters chosen were (fig.13), in the case of the 28mm, much more acceptable final results were had. It is necessary to make note of the fact that, above all, the value of the remainders on the

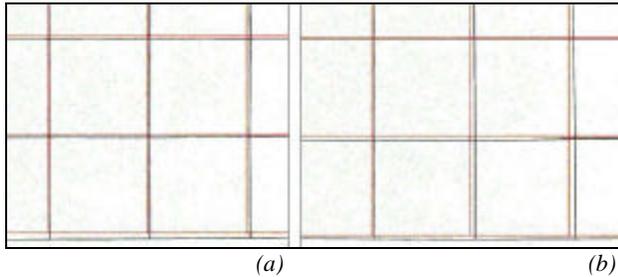


fig.13 - 20 points with 16 parameters (a), 20 points with 20 parameters (b)

Calcolo dei parametri della trasformazione

numero	X	Y	res X	res Y	res X	res Y
5	695.2928	875.3866	-3.7072	-0.6134	-1.6563	-0.2740
6	861.5078	875.2799	-3.4922	-0.7201	-1.5919	-0.3283
8	1193.9378	875.0666	4.9378	2.0666	2.6135	1.0958
10	196.7670	707.5546	1.7670	-0.4454	0.8081	-0.2037
16	1195.4935	709.6070	2.4935	-0.3930	1.1682	-0.1841
17	30.1923	538.6117	4.1923	-1.3883	1.8936	-0.6271

Calcola diretta Calcola inversi N° parametri: 0

Disegna Colore scala residui: 10

SigmaZeroQ

par	valore	varcov
1	30.4338192858	3.0053173824
2	-0.0120273300	0.0029388834
3	16.6214995068	0.0014122476

5.981110

fig.14 - a window of GPT where it's possible to see that the residual are higher for reflex cameras

control points is surely greater (fig.14) with respect to the case of the digital cameras, but for the non-metrical reflex it is necessary to take into consideration the systematic errors due to the non planarity of the film and to the use of the scanner.

For these cameras, the correction of the deformations introduced by the distortion is had only through the used of an elevated number of control points (20-25 points) and developing the polynomial until the terms of the third degree (fig.15a); halving the number of the points, the images are resampled with much more elevated deformations and therefore they do not supply metrically valid results. (fig.15b)



fig.15a

fig.15b

SOME COMMENT AND NEXT DEVELOPMENTS

The described procedure is still in a spermental phase and certainly is to be improved. The greatest care in the photogrammetric project, both for the exterior and interior orientation parameters. Non-metric cameras, just because they don't have a settled (stable) interior system of reference, need to adopt some particular solutions such as taking photograms with the optical system at the iperfocal position, to obtain an unknown but stable focal length. About the image coordinates system, problems can be resolved easily in the case of digital cameras, where the reference system is that of the image. In fact the application of polynomial forms to correct distorsions introduced by camera lens was satisfied in the case of digital cameras.

About the non-metrical reflex, the use of lens with high distortions (17mm) and the difficulty to define a stable reference system of the film don't give the wanted results. The spermentation work will go on with the attempt to fix the image coordinates system on the non metric photograms using the external edges of the photogram, which intersection allows to single out some sufficiently stable points.

BIBLIOGRAPHY

- Kenneth R. Castleman, *Digital Image Processing*, Prentice Hall International, London, 1996.
- Keith C. Clarke, *Analytical and Computer Cartography*, Prentice Hall International, London, 1995.
- Fangi Gabriele, *Notes in photogrammetry*, Clua Edizioni, Ancona, 1997.
- Andrew S.Glassner, *Principles of digital image processing*, Morgan Kaufmann, S.Franisco, 1995.
- Cliff Greve, *Digital Photogrammetry*, ASPRS, 1996.
- Karl Kraus, *Photogrammetry*, Ferd Dumlmlers Verlag, Bonn, 1993
- Marc van Kreveld..., *Algorithmic Foundation of Geographic Information System*, Springer, Berlin, 1997.
- Pietro Melli, *The digital elaboration of images*, Franco Angeli, Milano 1991;
- William K. Pratt, *Digital Image Processing*, John Wiley & Son, New York, 1991.
- Attilio Selvini, *Elements of photogrammetry*, CittàStudi, Milano 1994;
- Scott E. Umbaugh, *Computer Vision and Image Processing*, Prentice Hall International, London, 1998
- Fabio Crosilla, Francesco Guerra, Domenico Visentini, *Un possibile sviluppo analitico della D.L.T. e sua applicazione per il rilievo dell'architettura*. Bollettino SIFET n.1/1993.
- Francesco Guerra, *Studio analitico e metodologico della fotogrammetria con camere non metriche per il rilievo architettonico*, tesi di laurea 1988/89