A BIRD’S EYE VIEW ON SWITZERLAND IN THE 18TH CENTURY: 3D RECORDING AND ANALYSIS OF A HISTORICAL RELIEF MODEL

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ABSTRACT:

The relief of Franz Ludwig Pfyffer, constructed between 1762 and 1786, is considered the world’s oldest large landscape model and a pioneering work of Swiss cartography. Based on Pfyffer’s own measurements, the 6.6 x 3.9 m² big masterpiece represents a fascinating bird’s eye view of the Alps. The relief is not only an extraordinary topographic but also a cultural achievement: It has once been an attraction for numerous representatives of European social and scientific life. This paper reports about the procedures for the digital recording and quantitative evaluation of the Pfyffer’s relief, which are part of a broad interdisciplinary research. For the first time, the results of the photogrammetric 3D reconstruction and accuracy analysis of the whole object are presented. The DTM, the orthoimage and the complete vector data set were archived for the documentation of the cultural heritage. For the reason of project publicity, the derivation of high quality visualization products such as flyovers and interactive models was an important issue, too. In the second part the paper describes methods for the comparison of the virtual historical model with current map information. The procedures are based on a 9-parameter transformation of identical points between two three-dimensional datasets. In particular, a new procedure for the transformation of historical height and image data has been developed and evaluated. Furthermore, a novel method for analysis of the lake contours accuracy in the old relief or in the old map respectively is suggested. The procedures enable visualization of relief distortions (including a distortion grid), overlays with current vector data as well as direct comparison of the digital terrain models. The results of the analysis bring new surprising information concerning geometric accuracy, metric parameters and building strategy of the Pfyffer’s relief.

1. INTRODUCTION

Old maps, charts and 3D models have traditionally been a subject of study by the historians. These masterpieces combining art and a great knowledge of surveying and cartography currently receive deserved attention of scientists as well. In particular, photogrammetry and digital image processing are used to extract metrical and semantic characteristics of historical maps (for example see Fuse, 1998; Shimizu, 1999; Baletti, 2000). This paper addresses the three-dimensional aspect of historical cartography and focuses on the image-based reconstruction and geometrical evaluation of one of the most remarkable landscape models in history – the relief of Franz Ludwig Pfyffer.

Historical reliefs can be considered a direct ancestor of present virtual models. At the end of the 18th century, the bird-eye view at a landscape in the form of a relief was a special, unique experience comparable with our today’s enthusiasm for virtual reality. The fascination of exploring and analysing the third dimension of the mountainous area of Switzerland - once surely stimulation for the construction of Pfyffer’s relief - has motivated this research.

2. PFYFFER’S RELIEF OF SWITZERLAND

Lieutenant general Franz Ludwig Pfyffer von Wyher (1716-1802) devoted 20 years of his life to the construction of a relief of Central Switzerland (Figure 1). The result of his surveying and cartographic work is a 6.7 x 3.9 m² big model of Lake Lucerne and neighbouring cantons with a maximal elevation range of about 30 cm. It displays as much as 1/10 of the country in a scale of about 1:11'500. Finished in 1786, Pfyffer’s relief attracted numerous visitors from all over Europe to Lucerne: “One of the most impressive sights is to be seen in the general Pfyffer’s flat; namely, a topographic representation of a big part of the federation, highly admired by experts. It is up to the last detail correct and contains all the mountains, lakes, rivers and villages, as well as each cottage, bridge and road; even every cross is accurately and clearly depicted.” (Leu, H. J., 1788). This masterpiece is a significant improvement of existing maps at that time and as such it served as a basis for several printed works issued at the end of 18th century. A detailed study of Pfyffer’s relief probably played an important role in the victorious battle of French troops against the Russian field marshal Alexander Suvorov in the mountainous St. Gotthard region in 1799. As a strategic cartographic object, the French commander Napoleon Bonaparte planned to buy the relief in 1805 (Imhof, 1981). Fortunately, at that time better map information was already available and the model stayed Swiss property. Today it is on display in the Gletschergarten Museum in Lucerne as the world’s probably oldest and very detailed large landscape model.

Figure 1. A view on the 26 m² large Pfyffer’s relief
Within a project supported by the Swiss National Science Foundation and local cultural authorities, the relief and its constructor Franz Ludwig Pfyffer are subjects of an interdisciplinary research. The Institute of Geodesy and Photogrammetry of ETH Zurich is involved with following tasks:

- 3D reconstruction of the relief for the documentation of the cultural heritage.
- Analysis of the relief accuracy for the purpose of research in the history of cartography – a comparison of the reconstructed virtual relief with current map information (digital maps and terrain models). This work also includes analysis of the old maps of the region which were created at the end of 18th century.
- Investigation on Pfyffer’s surveying and model constructing procedures.

3. 3D RECONSTRUCTION AND VISUALIZATION

The project objectives require a high quality 3D model of Pfyffer’s relief in terms of accuracy. The overall height difference between the reconstructed model and the original may amount to a maximum of 1 mm. Represented by around 11.5 m in reality, this value approximately corresponds to the accuracy of available national DTM in alpine regions. Thus, a comparison of the historical and present terrain models in the analytical project phase (see Chapter 4) can bring reasonable results.

3.1 Data acquisition

The relief is situated in a narrow cellar room without natural light on a table about 65 cm over the ground. These conditions make the image acquisition as well as control point measurement rather difficult. From a construction platform fixed at the height of 185 cm above the relief surface, 87 analogue and 50 digital “aerial” coloured images were taken, both providing stereo overlap for the whole object (Figures 2 and 3, Table 1). The reason for such a double-block image acquisition was to make sure that the image quality will match the expectations because no new acquisition would have been possible any more. Both cameras were calibrated using a close-range photogrammetric reference field. To avoid disturbing specular reflection of the shiny relief surface, spotlights in a combination with dispersion umbrellas were used for indirect illumination. In spite of this, particularly the digital images show up rather big reflection and worse radiometric quality than their sensitive film-based counterparts.

The three-dimensional position of 40 marked control points temporarily placed onto the relief surface was determined by theodolite measurements in a local coordinate system with an accuracy of 0.09 mm. Considering the unfavourable space conditions leading to flat and wide-angled rays this result is satisfactory and sufficient for further processing.

3.2 Photogrammetric processing and visualization

Within the previously finished pilot project the functionality of the automated DTM generation of several commercial digital photogrammetric stations (SocetSet, VirtuoZo, Image Station, Match-T) was tested. A comparison of the results with the manually measured reference showed that the automatic procedures do not bring required performance in this case. The height differences between automatically derived models and a manually measured reference rise up to 5-7 mm in a data set with a maximal elevation range of about 300 cm. However, using manual measurements, the theoretical accuracy of 0.3 mm (0.15‰ of the flight height above the object) can be expected. Main reasons for the problems of commercial image matchers are distinct height differences and the low contrast in the relief. As mentioned before, a very precise DTM is needed (height error of 1 mm in maximum). Thus, the analogue images were selected as a basis for the photogrammetric processing and we distorted to manual measurements in the main project phase.

Phototriangulation. The whole block was triangulated manually on an analytical plotter. The achieved accuracy of 7.62 microns corresponding to 0.17 mm in object space is good enough.

DTM generation. For the DTM generation 48 stereo pairs along the four strips as well as several additional image pairs in the mountainous area were measured manually. We used profile
measurements at a profile distance of 1 cm with additional breaklines, which gave us about 300'000 points in total. The grid calculation with an interpolation software system DTMZ developed at the Institute resulted to a regular raster of 1 cm grid width.

**Orthoimage generation.** The 87 coloured images were scanned on an UltraScan 5000 scanner of Vexcel Imaging with the resolution of 1270 dpi corresponding to a footprint of 0.4 mm in the object. This assures that even the smallest relief details are clearly visible. For orthoimage generation the system SocetSet of LH Systems was used. The resulted mosaic covers the whole DTM area and uncompressed has a file size of about 306 MB.

**Extraction of 3D vector data.** In order to get a vector data set for the accuracy analysis of Pfyffer’s relief, significant relief features like roads, rivers, lakes and settlements were captured three-dimensionally in a manual mode on the analytical plotter.

**Texture mapping and visualization.** Mapping the orthoimage onto the DTM, a variety of visualization products were derived (Figure 4): anaglyph images, interactive VRML models and flyovers. A virtual flight over the reconstructed relief or an online navigation in the model is nowadays as fascinating as in the age of Franz Ludwig Pfyffer the relief itself.

![Figure 4. A view of reconstructed model of the Pfyffer’s relief (created using the software Skyline)](image)

The complete digital data set of Pfyffer’s relief was archived at the Kulturgüterschutz of Lucerne for the documentation of cultural heritage. In case of the damage of the relief or its parts, the precise digital data can be used for physical reconstruction of the original.

4. **ACCURACY ANALYSIS OF THE RELIEF**

In order to determine the accuracy of Pfyffer’s relief, the reconstructed model must be compared with current map information. Considering relief distortion in all three directions, various methods of spatial geometric transformations have been implemented and solutions are proposed in this section. In particular, a comparison of the historical terrain model with the current data represents a new problem in the historical cartography: rather than rectifying and georeferencing a planar old map, the 3D model has to be analysed. An important goal of accuracy analysis is a good visual presentation of the work; the methods and results must be easily understandable for project partners – historians with less technical background. As programming languages C and Matlab are used. The input data and all the results are georeferenced, maintained and visualised in ArcView GIS.

Two data sets are used for the comparison:

1. **Historical data in the local coordinate system: DTM of 1 cm grid width, orthoimage of 0.5 mm footprint (13'120 x 7780 pixel) and structured 3D vector data.** The data set covers the whole relief area (6.7 x 3.9 m²).
2. **Current data in the Swiss national coordinate system: DTM of 25 m grid width, digital map 1:25'000 with 2.5 m footprint and structured 2D vector data VECTOR25.** The data set covers an area of about 96 x 87 km².

The accuracy analysis of Pfyffer's relief is based on a number of identical points selected according to principles of historical cartography. The procedures described in the following include definition and transformation of identical points, transformation of image, height and vector data, visualization of the results, analysis of lake contours accuracy and the DTM comparison.

**4.1 Definition and transformation of identical points**

Identical points are objects in Pfyffer’s relief, which in reality have existed for centuries and which can reliably be found in both data sets: churches, crossings, mountain peaks, bridges etc. For each point, 3D coordinates x, y, z of historical data in the local system and X, Y, Z of current data in the national coordinate system are stored. Additionally, identical points are assigned to one of three categories according to their estimated reliability. In co-operation with historians, overall 221 well distributed identical points were defined (Figure 5).

![Figure 5. A chapel on a lake island as a reliable identical point, left in Pfyffer’s relief, right in the 1:25'000 map](image)

To determine the absolute accuracy of Pfyffer’s relief, identical points have been transformed using a spatial transformation with 9 degrees of freedom - 3 shifts, 3 rotations and 3 scales, which proved to be particularly suitable for old relief models (see Niederoest, 2002a):

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
= 
\begin{pmatrix}
d_x \\
d_y \\
d_z
\end{pmatrix} +
\begin{pmatrix}
m_x \\
m_y \\
m_z
\end{pmatrix}
R(\alpha, \beta, \gamma)
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

where \( x, y, z = \) coordinates in historical data set
\( X, Y, Z = \) coordinates in current data set
\( d_x, d_y, d_z = \) shifts in three coordinate directions
\( m_x, m_y, m_z = \) scales in three coordinate directions
\( \alpha, \beta, \gamma = \) rotation angles
\( R = \) rotation matrix

R = rotation matrix

\[\begin{bmatrix}
\alpha & \beta & \gamma \\
-\beta & \alpha & 0 \\
-\gamma & 0 & \alpha
\end{bmatrix} \]
The results of the transformation are summarized in Table 2. The 221 tested points deviate from the current map in average of absolute values about 387 m in the plane and 76 m in the height, corresponding to 3.3 and 0.7 cm within the object. Considering the conditions under which the relief was constructed – a definitely unfavorable age for landscape exploration and surveying, poor available maps and a very large area modelled in detail – Franz Ludwig Pfyffer’s achievement is even from today’s point of view admirable. The more detailed cartographic-historical interpretation of the results as well as a comparison of the relief with the old maps of that period can be found in (Niederoest, 2002b).

<table>
<thead>
<tr>
<th>Scale</th>
<th>In X: 1:11'688 (±29)</th>
<th>In Y: 1:11'414 (±21)</th>
<th>In Z: 1:11'136 (±672)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declination from the north</td>
<td>30.3° (±0.001)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Metric parameters of the Pfyffer's relief (results of the 9-parameter transformation of identical points with weighted observations)

### 4.2 Transformation of image, height and vector data

When analysing the accuracy of historical relief models (and old maps respectively), we are not only interested in their metric parameters based on the transformation of identical points. In addition to procedures presented in the previous section, the transformation of the image, height and vector data to the modern coordinate system can be of great use for the research in the history of cartography. The complete historical data set transformed and georeferenced in this way allows overlays with current data and, in case of historical relief models, simple comparison of terrain models. Two approaches were developed for this purpose: transformation of historical data using global parameters and a mesh-wise transformation using local parameters. In (Niederoest, 2002a) was shown that the latter transformation, an often used method for the geometrical correction of historical maps (Fuse, 1998; Shimizu, 1999; Baletti, 2000) is less suitable for analyses of historical maps and relief models than the one using global parameters.

In the following the transformation of historical image, height and vector data to the modern coordinate system using global parameters is described. The procedures (a) and (c) can be in an analogue manner applied for analysis of (planar) old maps. In this case, use formulas of appropriate two-dimensional transformation instead of Equation 1 and do not consider algorithm steps dealing with the height information.

Using the previously estimated 9 parameters of the general spatial transformation of identical points \( d_x, d_y, d_z, m_x, m_y, m_z, \alpha, \beta, \gamma \) the orthoimage, DTM and the vector data set are transformed to the national coordinates as follows.

(a) Transformation of the image data. For each pixel \( U, V \) of the new texture image an algorithm similar to the orthoimage rectification procedure is applied (indirect method to avoid creation of holes):

1. Using georeferencing information (pixel footprint and coordinates of image corners), calculate \( X, Y \)
2. Find \( Z' \) value of the nearest DTM node of the transformed historical data set (nearest neighbour interpolation)
3. Calculate \( x, y \) for \( X, Y, Z' \) as an inversion of Eq. 1
4. Calculate corresponding pixel \( u, v \) of historical image using georeferencing information
5. Take grey values of \( u, v \) as transformed radiometric information for \( U, V \) pixel of the new image

As the image data of historical relief models - or the old maps respectively - are usually very large, the algorithm allows user-defined tiling of the transformed data set. This is particularly useful in the case when the dataset after the transformation declines from the north significantly. The seamless division of one huge image to several georeferenced tiles helps to optimize the performance of visualization.

(b) Transformation of the height data. The transformed DTM of the historical relief is primarily needed for the comparison with the current height model (a raster of 25 m grid width). To avoid errors from the double interpolation at this step, the DTM of the Pfyffer’s relief is transformed point-by-point applying the Equation 1, resulting to an irregular 3D point cloud. This raw data serves as a basis for the DTM comparison (Section 4.5). For the visualization purposes a regular raster of 25 m grid width is calculated using already mentioned package DTMZ which performs Delaunay triangulation and finite element interpolation.

(c) Transformation of the vector data. Each layer of the three-dimensional historical vector data set is transformed point-by-point using the Equation 1. The transformed points are reconnected to form the same linear features as in the original data set.

As a result, all DTM, orthoimage and vector features of the historical data set are in national coordinates now and thus can be overlaid with current map information (Figure 6), used for the visualization of the results (section 4.3) and for the further analysis (sections 4.4 and 4.5).

![Figure 6. A part of the orthoimage of Pfyffer’s relief (Lake Lucerne) in national coordinate system overlaid with current lake outlines (dark blue polygons).](image-url)
4.3 Visualization of the accuracy analysis

After performing the 9-parameter transformation, the residuals on identical points can be depicted and analysed. The visualization is not only performed in the original historical data set, but also in the data set previously transformed to the national coordinate system. This procedure allows the correction of obviously wrong defined points in the historical data set as well as the check of the visualization methods, particularly of the correct orientation of differences (Figure 7).

Figure 7. Differences on identical points in XY direction, left in original data set, right in the transformed data set; their comparison allows for the check of visualization procedures.

In order to obtain a view of the relief distortion as a whole, a so called distortion grid is generated (Figure 8). It represents the depiction of the current kilometre or geographic grid in the old map (e.g. Beineke, 2001). The distortion grid is based on the Delaunay triangulation on identical points: in the intersections of the current grid with the triangle sides the residuals in X and Y directions are linearly interpolated. The interpolated residuals are connected to form the horizontal and vertical lines. For the visualization of the distortions in the height, contour lines of the same height difference are interpolated in a similar way.

Figure 8. The distortion grid of the Pfiffer's relief in XY-direction (red lines). The light grey dotted lines represent the current 2-km grid.

4.4 Analysis of the accuracy of lake forms

Once transformed to the national coordinate system, the complete historical data set is georeferenced and the position and shape of its features such as roads, rivers, lakes etc. can be compared with the current reference data. The lakes represent one of the main characteristics of the area of Central Switzerland and as such they always had been objects of particular interest for old map makers. Therefore an investigation on the accuracy of lakes representation in the Pfiffer's relief was performed (in 2D).

For this purpose the measures originally developed for the quality assessment of building reconstruction (Niederoest, M., 2003) were used (Equation 2). The common lake area in respect to the current reference data (relative intersecting area) gives useful information on how good the two areas fit. The total relative shape dissimilarity represents the not common lake areas divided by the reference area; the ratio which should be small.

\[
\begin{align*}
\text{Relative intersecting area} & = \frac{\text{Ref} \cap \text{Old}}{\text{Ref}} \\
\text{Lake in the reference, but not in the relief} & = \text{Ref} \setminus \text{Old} \\
\text{Lake in the relief, but not in the reference} & = \text{Old} \setminus \text{Ref} \\
\text{Total relative shape dissimilarity} & = \frac{\text{Ref} \setminus \text{Old} + \text{Old} \setminus \text{Ref}}{\text{Ref}} \\
\end{align*}
\]

(2)

where \( \text{Ref} = \) lake area in the current reference data, \( \text{Old} = \) lake area in the Pfiffer's relief

The meaning of Equation 2 is illustrated in Figure 9b. It is visible that the lake in the historical relief is rather good if only the shape would be considered, but the shift between the two lake depictions is very big. Therefore for the area comparisons in the history of cartography it is suggested to treat the location and form of the feature independently: before computing the measures from Equation 2, the historical data is shifted to the centre of gravity of the reference (Figure 9c). Additionally, two new measures are defined:

\[
\begin{align*}
\text{Distance between the centres of gravity} & = \sqrt{(X_{\text{Ref}} - X_{\text{Old}})^2 + (Y_{\text{Ref}} - Y_{\text{Old}})^2} \\
\text{Average shape difference} & = \frac{\text{Ref} \setminus \text{Old} + \text{Old} \setminus \text{Ref}}{\text{Ref}_{\text{perim}}} \\
\end{align*}
\]

(3)

where \( X_{\text{Ref}}, Y_{\text{Ref}} = \) centre of gravity of the reference lake, \( X_{\text{Old}}, Y_{\text{Old}} = \) centre of gravity of the lake in the relief, \( \text{Ref} = \) lake area in the current reference data, \( \text{Old} = \) lake area in the Pfiffer's relief, \( \text{Ref}_{\text{perim}} = \) lake perimeter in the reference data

Figure 9. Quantitative analysis of the lake contours accuracy. (a) The lake in Pfiffer's relief overlaid with the reference, (b) Red: reference and relief (\( \text{Ref} \cap \text{Old} \)), blue: reference without relief (\( \text{Ref} \setminus \text{Old} \)), green: relief without reference (\( \text{Old} \setminus \text{Ref} \)), (c) The same analysis including the shift of the relief data to the reference centre of gravity.
The distance between the centres of gravity of 14 lakes in the Pfyffer's relief and in the current reference data is very diverse and reaches the values from 129 m to 1460 m. However, when the wrong position of the lakes is not considered (after the translation to the reference centre of gravity), the common area in respect to the reference data is only in two cases less than 60%. The total relative shape dissimilarity also shows rather a good result: only in three cases this ratio is more than 60%. Useful information about the representation of the lake form in the historical data is given by the not common areas divided by the reference perimeter (average shape difference). This measure reaches the value of less than 200 m by the 80% of the analysed data, what is significantly better than the relief parameters listed in Table 2. This fact underlines the good local accuracy of the relief features after their absolute position had been corrected.

4.5 Comparison of the height models

The DTM of Pfyffer's relief transformed to the national coordinate system is compared with the current DTM. The procedure is performed as a comparison of the reference regular raster and the irregular point cloud of the historical data (nearest neighbour interpolation). According to expectations the biggest height differences are located in the southern alpine part of the relief (Figure 10). The unusually big values in the histogram should not be understood as errors in Pfyffer's height measurement: they can rather be explained by the wrong position of several mountains. After removing the worst 2% of the data set as outliers (white spots in the lower part of the Figure 10), the average height difference in absolute values reaches 177 m (corresponding to 1.6 cm in the relief). This is a very respectable number for area-wide height measurements at the end of 18th century, because the maps at that time usually do not contain any height information.

5. CONCLUSIONS

On the example of the relief model of Switzerland, constructed by Franz Ludwig Pfyffer in 1762-1786, methods for 3D reconstruction and accuracy analysis of historical reliefs were shown. An interesting feature of the photogrammetric part of the project is the unusual combination of aerial and close-range applications: a big landscape mapping process is performed in an artificial environment of the cellar room of a museum. Since a very precise DTM is required, manual measurements of the surface were applied. The complete digital data set was archived for the documentation of the cultural heritage.

For the first time the relief of Franz Ludwig Pfiffer has been quantitatively evaluated. As there is an absolute lack of written documents concerning the relief construction, the applied procedures represent the only possible way of the exploration of this chapter of the Swiss history. The a-posteriori standard deviation of spatial vector on identical points, reaching 3.2 cm within the 26 m² big relief shows a surprising accuracy of the relief constructed long time before the first Swiss national triangulation network was established. In particular, the precise representation of the third dimension is remarkable for that time. The presented approaches extend the procedures used in the history of cartography to the vertical coordinate direction. In addition, some innovative methods for the analysis of polygonal features of the old maps and relief models as well as for the visualization are suggested.

REFERENCES


